CA CCSS — 8th Grade: Geometry
Math Unit: Understand and apply the Pythagorean Theorem.
By Michelle Cooley

8.G.6. Explain a proof of the Pythagorean Theorem. [*For all right triangles, the sum of the squared legs equals the hypotenuse squared ($a^2 + b^2 = c^2$)*];
Explain its converse: [*If the hypotenuse squared is equal to the sum of the squared legs (c2 = a2 + b2), the triangle is a right triangle*].
8.G.7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
8.G.8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

ENGAGE: The Humans Behind the Theorem
Ask if a student can come up to write and explain the Pythagorean theorem. Ask peers if they would like to contribute to this explanation. The T will clarify points as needed.

Where did this theorem come from? Who is Pythagorus? Ask students to put heads together in table groups to study this article: https://www.biographyonline.net/spiritual/pythagoras.html. Ask them to take notes in their math journals, with these questions as prompts:

1. Where and when was Pythagoras born?
2. Why did he travel so much? (He was on a quest for all knowledge.)
3. What organization did he lead?
4. Did Pythagoras discover the Pythagorean theorem?
5. What is the theorem?
6. To whom or what did the Pythagoreans pray?

Initiate a class discussion, answering questions and asking students about their thirst for knowledge? Do we travel long distances today to obtain knowledge?

EXPLORE: 8.G.7. The Theorem, and Pythagorean Triples (positive integer solutions). Have students engage in drawing and solving right triangles with the smallest two Pythagorean triples (define): 3,4,5 and 5,12,13. Ask the students to scale up the common triples, to see if they can easily create more triples. They can use the distributive property of multiplication.

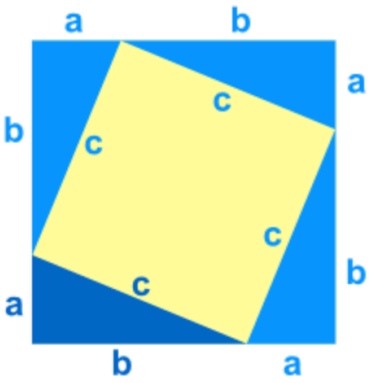
Example: $(3n)^2 + (4n)^2 = (5n)^2$ Blue Chart: Not “scaled up" or “primitive” triples

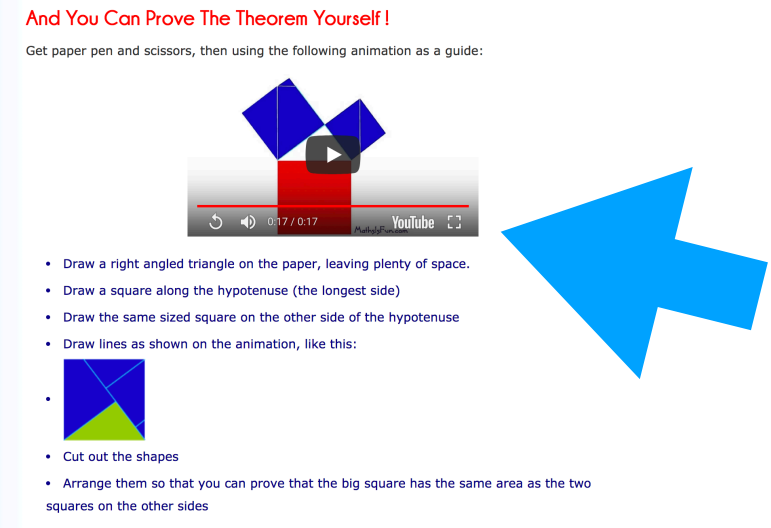
List of the First Few

Here is a list of the first few Pythagorean Triples (**not** including "scaled up" versions mentioned below):

(3,4,5)	(5,12,13)	(7,24,25)	(8,15,17)
(9,40,41)	(11,60,61)	(12,35,37)	(13,84,85)
(15,112,113)	(16,63,65)	(17,144,145)	(19,180,181)
(20,21,29)	(20,99,101)	(21,220,221)	(23,264,265)
(24,143,145)	(25,312,313)	(27,364,365)	(28,45,53)
(28,195,197)	(29,420,421)	(31,480,481)	(32,255,257)
(33,56,65)	(33,544,545)	(35,612,613)	(36,77,85)
(36,323,325)	(37,684,685)	... infinitely many more ...	

https://www.mathsisfun.com/geometry/pythagorean-theorem-proof.html





EXPLAIN:

8.G.6. Proof: Ask students to prove the theorem using the above-left blue and yellow **squares**. They can work as table groups. HINT: Find the area of the blue square in two ways. We know that the triangles are right triangles because each has a 90^a angle at the corners of the blue square.

$(a+b)(a+b)$ = $c^2 + 4(ab/2)$ Area of the Blue Square = Area of Blue Triangles + Yellow Square

$c^2 + 2ab$

$a^2 + 2ab + b^2$ = $c^2 + 2ab$

Subtract 2ab (area of blue triangles) from both sides of the equation.
(subtraction property of equality).

$a^2 + b^2$ = c2 Area of the Yellow Square AND the Pythagorean theorem!

A second proof with a manipulative puzzle (see the giant arrow) — This video
https://www.youtube.com/watch?time_continue=1&v=_87RbSoELW8
from mathsisfun is visually engaging. The students can each be given two congruent squares, red and blue. These represent c^2. The students are given a white right triangle; its hypotenuse is the side length of the squares. The blue and green square above shows students how to cut up the blue square so that the pieces can be formed into squares measuring a^2 and b^2 . These smaller blue squares are then aligned along the legs of the right triangle. The red, along the hypotenuse.

The Converse of the Proof: This comes in handy when we don't know if a triangle is a right triangle. If the squared hypotenuse is equal to the sum of the squared legs, then the triangle is a right triangle. We can also identify obtuse and acute triangles with the converse. If c^2 is greater than $a^2 + b^2$, then the triangle is obtuse. If c^2 is less than $a^2 + b^2$, then the triangle is acute.

ELABORATE & Develop Fluency: 8.G.7.

Guided practice using the PyTh: solving unknown lengths of right triangles, and determining types of triangles by angle (right, obtuse, acute). Real life situations will be used, i.e., ladders, heights of trees by shadows, and fencing needs. BRING IN A REAL LADDER. Lean it against the classroom wall. Apply the Pythagorean theorem by measuring the legs and hypotenuse of the right triangle formed. Square each side. Do the numbers abide by the Theorem? Crunch those numbers!

Review simplifying square roots. Practice. Right triangles may have sides that are not integers.

Group/Independent Activity: Ask the students to use a map, selecting a continent to explore. They will plan a three-stop trip, flying by helicopter, returning to the first stop at the end. They must figure out how much gasoline they will need for the trip, and they must calculate the Pythagorean theorem, in miles, to confirm what type of triangle is formed by their travel plans. The students will share their results by Padlet, attaching a diagram of their map, starred sites, and the type of triangle formed as proved by the PyTh. Random reward: given to the student or group that plans a linear trip. Hahaha. They saved math energy.

8.G.8
Using the coordinate system and the PyTh, find the distance between two points. The T will model. Challenge the students to complete a problem as table groups. Discuss. Have a volunteer table group explain their answer.

PyTh Independent Practice/Math Centers/Task Cards/Games: Students will become fluent in finding distances between two points on the coordinate system. They will practice proofs, the converse, and solving unknown sides of right tri's.

EVALUATE
Informal Summative Assessment, as time allows: Kahoot Game: PyTh w/History
Formal Summative Assessment will include:
1. explaining a proof of the PyTh and its converse.
2. solving the unknown sides of right triangles with two given sides.
3. solving the distance between two points on coordinate system, using PyTh.